Published by Institute of Physics Publishing for SISSA

Received: *June 8, 2007*
Accepted: *July 16, 2007*
Published: *July 19, 2007*

AdS_3, black holes and higher derivative corrections

Justin R. David, Bindusar Sahoo and Ashoke Sen

Harish-Chandra Research Institute
Chhatnag Road, Jhusi, Allahabad 211019, India
E-mail: justin@mri.ernet.in, bindusar@mri.ernet.in, sen@mri.ernet.in

Abstract: Using AdS/CFT correspondence and the Euclidean action formalism for black hole entropy Kraus and Larsen have argued that the entropy of a BTZ black hole in three dimensional supergravity with (0,4) supersymmetry does not receive any correction from higher derivative terms in the action. We argue that as a consequence of AdS/CFT correspondence the action of a three dimensional supergravity with (0,4) supersymmetry cannot receive any higher derivative correction except for those which can be removed by field redefinition. The non-renormalization of the entropy then follows as a consequence of this and the invariance of Wald's formula under a field redefinition.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence.

BTZ solution describes a black hole in three dimensional theory of gravity with negative cosmological constant [1] and often appears as a factor in the near horizon geometry of higher dimensional black holes in string theory [2]. Furthermore the entropy of a BTZ black hole has a remarkable similarity to the Cardy formula for the degeneracy of states in the two dimensional conformal field theory [3]. For these reasons computation of the entropy of BTZ black holes has been an important problem, both in three dimensional theories of gravity and also in string theory. Initial studies involved computing Bekenstein-Hawking formula for BTZ black hole entropy in two derivative theories of gravity. Later this was generalized to higher derivative theories of gravity [4 – 10], where the lagrangian density contains arbitrary powers of Riemann tensor and its covariant derivatives as well as gravitational Chern-Simons terms [11], both in the Euclidean action formalism [12] and in Wald's formalism [13 – 16].

While the above mentioned formalism tells us how to calculate the entropy of a BTZ black hole for a given action with arbitrary higher derivative terms, it does not tell us what these higher derivative terms are. It was however argued by Kraus and Larsen [5, 6] using AdS/CFT correspondence that if the three dimensional theory under consideration has at least (0,4) supersymmetry then the entropy of a BTZ black hole of given mass and angular momentum is determined completely in terms of the coefficients of the gravitational and gauge Chern-Simons terms in the action and hence does not receive any higher derivative corrections. This result is somewhat surprising from the point of view of the bulk theory, since for a given three dimensional theory of gravity the entropy does have non-trivial dependence on all the higher derivative terms. Thus one could wonder how the dependence of the entropy on these higher derivative terms disappears by imposing the requirement of (0,4) supersymmetry.

In this note we shall propose a simple explanation for this fact: (0,4) supersymmetry prevents the addition of any higher derivative terms in the supergravity action (except those which can be removed by field redefinition and hence give an equivalent theory). Our argument is based on the following observation. In AdS/CFT correspondence the boundary operators dual to the fields in the supergravity multiplet are just the superconformal currents associated with the (0,4) supersymmetry algebra. The correlation functions of these operators in the boundary theory are determined completely in terms of the central charges c_L, c_R of the left-moving Virasoro algebra and the right-moving super-Virasoro algebra. Of these c_R is related to the central charge k_R of the right-moving SU(2) currents which form the R-symmetry currents of the super-Virasoro algebra and hence to the coefficient of the Chern-Simons term of the associated SU(2) gauge fields in the bulk theory. On the other hand $c_L - c_R$ is determined in terms of the coefficient of the gravitational Chern-Simons term in the bulk theory. Thus the knowledge of the gauge and gravitational Chern-Simons terms in the bulk theory determines all the correlation functions of (0,4) superconformal currents in the boundary theory. Since by AdS/CFT correspondence [17] these correlation functions in the boundary theory determine completely the boundary S-matrix of the supergravity fields [18, 19], we conclude that the coefficients of the gauge and gravitational Chern-Simons terms in the bulk theory determine completely the boundary S-matrix elements in this theory.

JHEP07(2007)058

Now the boundary S-matrix elements are the only perturbative observables of the bulk theory. Thus we expect that two different theories with the same boundary S-matrix must be equivalent. (We shall elaborate on this later.) Combining this with the observation made in the last paragraph we see that two different gravity theories, both with (0,4) supersymmetry and the same coefficients of the gauge and gravitational Chern-Simons terms, must be equivalent. Put another way, once we have constructed a classical supergravity theory with (0,4) supersymmetry and given coefficients of the Chern-Simons terms, there cannot be any higher derivative corrections to the action involving fields in the gravity supermultiplet except for those which can be removed by field redefinition.[1] The non-renormalization of the entropy of the BTZ black hole then follows trivially from this fact. The complete theory in the bulk of course can have other matter multiplets whose action will receive higher derivative corrections. However since restriction to the fields in the gravity supermultiplet provides a consistent truncation of the theory,[2] and since the BTZ black hole is embedded in this subsector, its entropy will not be affected by these additional higher derivative terms.

Our arguments will imply in particular that the five dimensional supergravity action constructed in [21], after dimensional reduction on a sphere, must be equivalent to the three dimensional supergravity action given in eq. (2) below with the precise relationship between the various coefficients as given in eq. (4). This in turn would explain why the analysis of the black hole entropy given in [22, 23] (see also [24]) agrees with the expected result. We should caution the reader however that the field redefinition needed to arrive at the action given in (2) may not be invertible on all field configurations. For example if we take a Chern-Simons action and add to it the usual kinetic term for a gauge field then the kinetic term can be removed formally by a field redefinition. However the theory with the kinetic term has an extra pole in the gauge field propagator corresponding to a massive photon which is absent in the pure Chern-Simons theory. This happens because the field redefinition that takes us from the theory with the gauge kinetic term to pure Chern-Simons theory is not invertible on the plane wave solution describing the propagating massive photon. This however does not affect our argument as long as the field redefinition is invertible on slowly varying field configuration around the AdS_3 background. In this context we note that such field redefinitions are carried out routinely in string theory, e.g. in converting a term in the gravitational action quadratic in the Riemann tensor to the Gauss-Bonnet combination. The former theory typically has extra poles in the graviton propagator which are absent in the latter theory.

[1] Incidentally, since the correlation functions of the superconformal currents in the boundary theory, expressed in terms of k_R and c_L, are also not affected by inclusion of non-planar graphs, it follows that the supergravity action also does not receive any higher derivative quantum corrections. This however does not mean that BTZ black hole entropy is protected from quantum corrections; the global geometry of BTZ black hole is different from AdS_3, and due to this BTZ black hole entropy can receive correction from terms in the action which cannot be written as integrals of local Lagrangian density in three dimensions [20].

[2] In practice this means that there is no term in the action that contains a single power of a matter field and one or more powers of the supergravity fields. In the CFT living on the boundary this is reflected in the fact that a correlation function involving a single primary field other than identity and arbitrary number of superconformal currents vanishes.

JHEP07(2007)058

For completeness we shall now describe this unique (0,4) supergravity action and compute the entropy of a BTZ black hole from this action. The action was constructed in [25, 26] by regarding the supergravity as a gauge theory based on $\mathrm{SU}(1,1)\times\mathrm{SU}(1,1|2)$ algebra.[3] If Γ_L and Γ_R denote the (super-)connections in the $\mathrm{SU}(1,1)$ and $\mathrm{SU}(1,1|2)$ algebras respectively, then the action is taken to be a Chern-Simons action [30] of the form:

$$\begin{aligned}\mathcal{S} = & -a_L \int d^3x \left[Tr(\Gamma_L \wedge d\Gamma_L + \frac{2}{3}\Gamma_L \wedge \Gamma_L \wedge \Gamma_L \right] \\ & + a_R \int d^3x \left[Str(\Gamma_R \wedge d\Gamma_R + \frac{2}{3}\Gamma_R \wedge \Gamma_R \wedge \Gamma_R \right] ,\end{aligned} \tag{1}$$

where a_L and a_R are constants. Note that the usual metric degrees of freedom are encoded in the connections Γ_L and Γ_R [31]. Thus there is no obvious way to add $\mathrm{SU}(1,1)\times\mathrm{SU}(1,1|2)$ invariant higher derivative terms in the action involving the field strengths associated with the connections Γ_L and Γ_R. From this viewpoint also it is natural that the supergravity action does not receive any higher derivative corrections.

The bosonic fields of this theory include the metric G_{MN} and an SU(2) gauge field $\mathbf{A}_M$ $(0 \le M \le 2)$, represented as a 2×2 anti-hermitian matrix valued vector field. After expressing the action in the component notation and eliminating auxiliary fields using their equations of motion we arrive at the action

$$\mathcal{S} = \int d^3x \left[\sqrt{-\det G}\, \left[R + 2m^2\right] + K\, \Omega_3(\widehat{\Gamma}) - \frac{k_R}{4\pi}\, \epsilon^{MNP} Tr\left(\mathbf{A}_M \partial_N \mathbf{A}_P + \frac{2}{3}\mathbf{A}_M \mathbf{A}_N \mathbf{A}_P \right) \right] \tag{2}$$

where

$$\frac{1}{m} = \frac{1}{2}(a_R + a_L), \qquad K = \frac{1}{2}(a_L - a_R)\,, \tag{3}$$

$$k_R = 4\pi a_R = 4\pi \left(\frac{1}{m} - K \right) , \tag{4}$$

$\widehat{\Gamma}$ is the Christoffel connection constructed out of the metric G_{MN} and

$$\Omega_3(\widehat{\Gamma}) = \epsilon^{MNP} \left[\frac{1}{2} \widehat{\Gamma}^R_{MS} \partial_N \widehat{\Gamma}^S_{PR} + \frac{1}{3} \widehat{\Gamma}^R_{MS} \widehat{\Gamma}^S_{NT} \widehat{\Gamma}^T_{PR} \right] . \tag{5}$$

ϵ is the totally anti-symmetric symbol with $\epsilon^{012} = 1$. Note that although the action contains gravitational Chern-Simons term, there are no terms involving square of the Riemann tensor.

We shall now compute the entropy of a BTZ black hole in this theory. We begin by reviewing the result for BTZ black hole entropy in a general higher derivative theory of gravity. For this it will be enough to keep only the gravitational fields in the action, setting

[3] A different class of supergravity theories were constructed in [27 – 29] based on the supergroup $Osp(p|2;R) \times Osp(q|2;R)$, with the supercharges transforming in the vector representation of the R-symmetry group $\mathrm{SO}(p)_L \times \mathrm{SO}(q)_R$. Thus the corresponding boundary theories will have a different superalgebra and a different set of correlation functions, and our arguments cannot be used to relate the bulk action of these theories to the action given in (1).

JHEP07(2007)058

all other fields to zero. Let us consider a general gravitational action in three dimensions of the form:

$$S = \int d^3x \left[\sqrt{-\det G}\,\mathcal{L}_0^{(3)} + K\,\Omega_3(\widehat{\Gamma})\right] , \tag{6}$$

where $\mathcal{L}_0^{(3)}$ denotes an arbitrary scalar constructed out of the metric, the Riemann tensor and covariant derivatives of the Riemann tensor. A general BTZ black hole in the three dimensional theory is described by the metric:

$$G_{MN}dx^M dx^N = -\frac{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}{l^2\rho^2}d\tau^2 + \frac{l^2\rho^2}{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}d\rho^2 + \rho^2\left(dy - \frac{\rho_+\rho_-}{l\rho^2}d\tau\right)^2 , \tag{7}$$

where l, ρ_+ and ρ_- are parameters labelling the solution. Of these the parameters $\rho_\pm$ can be removed locally by a coordinate transformation, so that any scalar combination of the Riemann tensor and metric computed for this metric is a function of the parameter l only. We define

$$h(l) = \mathcal{L}_0^{(3)} , \tag{8}$$

evaluated in the background (7), and

$$g(l) = \frac{\pi l^3}{4}\,h(l) . \tag{9}$$

Then the following results hold (see e.g. [8]):

1. Equations of motion of the metric determines the value l_0 of l to be a solution to the equation

$$g'(l_0) = 0 . \tag{10}$$

2. The entropy of a BTZ black hole with mass M and angular momentum J is given by[4]

$$S_{BH} = 2\pi\sqrt{\frac{c_L q_L}{6}} + 2\pi\sqrt{\frac{c_R q_R}{6}} , \tag{11}$$

where

$$q_L = \frac{1}{2}(M-J), \qquad q_R = \frac{1}{2}(M+J) , \tag{12}$$
$$c_L = 24\,\pi\,(C+K) , \qquad c_R = 24\,\pi\,(C-K) , \tag{13}$$
$$C = -\frac{1}{\pi}g(l_0) . \tag{14}$$

3. The parameters $\rho_\pm$ are related to M and J via the relations

$$M \pm J = \frac{2\pi(C \mp K)}{l_0^2}(\rho_+ \pm \rho_-)^2 . \tag{15}$$

[4]It is worth emphasizing that since under a field redefinition of the metric $l \to f(l)$ for some function $f(l)$, l_0 is not invariant under a field redefinition. However $g(l_0)$, being the value of the function $g(l)$ at its extremum, is invariant under such a field redefinition.

JHEP07(2007)058

We shall now apply these results to the action given in (2). We get

$$h(l) = (-6l^{-2} + 2m^2)\,, \qquad g(l) = \frac{\pi}{4} l^3 \,(-6l^{-2} + 2m^2)\,, \qquad l_0 = \frac{1}{m}\,, \qquad C = \frac{1}{m} \tag{16}$$

and

$$c_L = 24\,\pi \left(\frac{1}{m} + K\right) = 24\,\pi\, a_L, \qquad c_R = 24\,\pi \left(\frac{1}{m} - K\right) = 24\,\pi\, a_R\,, \tag{17}$$

where in (17) we have used (3). Using (4) we get

$$c_R = 6\,k_R, \qquad c_L = 48\,\pi\, K + 6\,k_R\,. \tag{18}$$

Eqs. (11), (12), (18) give the desired expression for the entropy of a BTZ black hole in terms of the coefficients of the gauge and gravitational Chern-Simons terms. By our previous argument addition of higher derivative terms do not change this result as long as they respect (0,4) supersymmetry.

Since the crux of our argument has been the relationship between non-renormalization of the boundary S-matrix and the non-renormalization of the classical action, we shall now elaborate on this by examining how this works for the gauge sector of the theory. In this case the Chern-Simons theory has equation of motion $\mathbf{F}_{MN} = 0$ where

$$\mathbf{F}_{MN} \equiv \partial_{[M}\mathbf{A}_{N]} + [\mathbf{A}_M, \mathbf{A}_N] \tag{19}$$

is the gauge field strength. Any additional gauge invariant term in the action will involve the gauge field strength and hence will vanish when $\mathbf{F}_{MN} = 0$. The following standard argument then shows that such terms can be removed from the action using a field redefinition. Let S_0 denote the pure Chern-Simons action and S denote the full action including higher derivative terms. Then we can write

$$S = S_0 + \lambda \int d^3x\, F^a_{MN}(x)\, K^{aMN}(x)\,, \tag{20}$$

where K^{aMN} is a gauge covariant term and hence contains at least a single power of $F^{aMN}(x)$ or its covariant derivatives. λ is a small parameter that indicates that these additional higher derivative terms are small compared to the leading term for slowly varying fields. Now since

$$\frac{\delta S_0}{\delta A^a_M(x)} = c\,\epsilon^{MNP}\, F^a_{NP}(x)\,, \tag{21}$$

for an appropriate constant c, we can remove the second term in (20) to first order in λ by a field redefinition $A^a_M \to A^a_M - \frac{\lambda}{2c}\epsilon^{MNP} K^a_{NP}$. The resulting Lagrangian will have the form of a pure Chern-Simons action plus terms of order λ^2 and higher with each term containing at least two powers of F^a_{MN} or its covariant derivatives. Thus after integration by parts the action can be brought to the same form as (20), but now with λ replaced by λ^2 and K^{aMN} replaced by a different gauge covariant expression with higher number of derivatives. By repeating this process we can bring S to the form S_0 up to any given order in the expansion in powers of λ, i.e. any given order in the derivative expansion.

JHEP07(2007)058

We shall now see how the vanishing of the additional terms in the action for $\mathbf{F} = 0$ is related to the non-renormalization of the boundary S-matrix. For this we first review the computation of the boundary S-matrix from pure Chern-Simons theory. We begin by writing the Euclidean AdS_3 metric in the Poincare patch

$$ds^2 = \frac{l^2}{(x^0)^2} \left((dx^0)^2 + (dx^1)^2 + (dx^2)^2\right) , \tag{22}$$

and introduce complex coordinate z, integration measure d^2z and the δ-function $\delta^{(2)}(\vec{z})$ as follows:

$$z = x^1 + ix^2 , \qquad d^2z \equiv dx^1 dx^2 , \qquad \delta^{(2)}(\vec{z}) \equiv \delta(x^1)\delta(x^2) . \tag{23}$$

The gauge field action in the Euclidean space takes the form[5]

$$S_{\text{gauge}} = -i\frac{k_R}{4\pi} \int d^3x \epsilon^{MNP} Tr \left(\mathbf{A}_M \partial_N \mathbf{A}_P + \frac{2}{3} \mathbf{A}_M \mathbf{A}_N \mathbf{A}_P \right) + \frac{k_R}{2\pi} \int d^2z Tr(\mathbf{A}_z \mathbf{A}_{\bar{z}})|_{x^0=0} . \tag{24}$$

The last term in (24) is a boundary term needed to ensure the consistency of the theory [33, 25, 32]. The effect of this term is that while computing the variation of the on-shell action under a variation of the gauge fields, the result depends only on $\delta\mathbf{A}_z$ at the boundary $x^0 = 0$ and not on $\delta\mathbf{A}_{\bar{z}}$. Thus while deriving the equations of motion from the action using a variational principle, we fix the boundary condition only on $\mathbf{A}_z$ [32]. Let us denote by $\vec{z}$ the pair $(z, \bar{z})$ and let $I[\mathbf{A}_z^{(0)}]$ denote the value of the Euclidean action evaluated for an on-shell field configuration subject to the boundary condition

$$\mathbf{A}_z(x^0 = 0, \vec{z}) = \mathbf{A}_z^{(0)}(\vec{z}) . \tag{25}$$

Then according to AdS/CFT conjecture we have [18, 19]

$$\left\langle \bar{J}^{a_1}(\vec{z}_1) \cdots \bar{J}^{a_n}(\vec{z}_n) \right\rangle = (i\pi)^n \left. \frac{\delta}{\delta A_z^{(0)a_1}(\vec{z}_1) \cdots \delta A_z^{(0)a_n}(\vec{z}_n)} e^{-I[\mathbf{A}_z^{(0)}]} \right|_{\mathbf{A}_z^{(0)}(\vec{z})=0} , \tag{26}$$

where $\bar{J}^a(\vec{z})$ are the SU(2) currents of the CFT at the boundary and the A_M^a are defined through

$$\mathbf{A}_M = \frac{1}{2}\, i\, \sigma^a\, A_M^a , \tag{27}$$

σ_a being the Pauli matrices. Thus our task is to compute $I[\mathbf{A}_z^{(0)}]$. For this we first need to evaluate the gauge field configuration that satisfies the equation of motion $\mathbf{F}_{MN} = 0$ and the boundary condition (25). This is given by

$$\mathbf{A}_M(\vec{z}, x^0)\, dx^M = e^{-\Phi}\, d\, e^{\Phi} = d\Phi + \frac{1}{2}(d\Phi\Phi - \Phi d\Phi) + \cdots \tag{28}$$

[5] We shall follow the same sign and normalization convention as ref. [32]. There is an apparent difference in the overall sign of the Chern-Simons term, but this is related to the fact that in the x^0, x^1, x^2 coordinate system the boundary of AdS_3 is at the lower limit of x^0 ($x^0 = 0$) and we have chosen $\epsilon^{012} > 0$. As a result we need the $-$ sign in front of the Chern-Simons term to ensure that the variation of the on-shell action depends only on $\delta\mathbf{A}_z$ and not on $\delta\mathbf{A}_{\bar{z}}$ at the boundary.

JHEP07(2007)058

where

$$\Phi(\vec{z}, x^0) = \int d^2w K(\vec{z}, x^0; \vec{w})\, \mathbf{B}_z^{(0)}(\vec{w}), \tag{29}$$

$$K(\vec{z}, x^0; \vec{w}) = \frac{1}{\pi}\left[\frac{z-w}{(x^0)^2 + |z-w|^2}\right], \tag{30}$$

and $\mathbf{B}_z^{(0)}$ is chosen such that (28) satisfies the boundary condition (25). Eq. (30) gives

$$\lim_{x^0\to 0} \partial_z K(\vec{z}, x^0; \vec{w}) = \delta^{(2)}(\vec{z}-\vec{w}), \tag{31}$$

$$\lim_{x^0\to 0} \partial_{\bar{z}} K(\vec{z}, x^0; \vec{w}) = -\frac{1}{\pi}\frac{1}{(\bar{z}-\bar{w})^2}. \tag{32}$$

Using eqs. (28)- (31) we find that $\mathbf{A}_z(x^0 = 0, \vec{z})$ is equal to $\mathbf{B}_z^{(0)}(\vec{z})$ to first order in an expansion in a power series in $\mathbf{B}_z^{(0)}$. Thus to this order (25) is satisfied for $\mathbf{B}_z^{(0)} = \mathbf{A}_z^{(0)}$. The higher order contributions to $\mathbf{B}_z^{(0)}$ can be obtained by iteratively solving eq. (25) with the ansatz for $\mathbf{A}_M$ given in (28)- (30). The result is

$$\mathbf{B}_z^{(0)}(\vec{z}) = \mathbf{A}_z^{(0)}(\vec{z}) + \frac{1}{2\pi}\int \frac{d^2w}{(\bar{z}-\bar{w})}\left(\mathbf{A}_z^{(0)}(\vec{w})\mathbf{A}_z^{(0)}(\vec{z}) - \mathbf{A}_z^{(0)}(\vec{z})\mathbf{A}_z^{(0)}(\vec{w})\right) + \cdots \tag{33}$$

where $\cdots$ denote higher order terms. We can now substitute the solution given in (28)- (33) into (24) to evaluate the on-shell action $I[\mathbf{A}_z^{(0)}]$. Evaluation of the boundary contribution is straightforward. In evaluating the contribution from the Chern-Simons term we first use the equation of motion to express it as

$$i\frac{k_R}{12\pi}\int d^3x \epsilon^{MNP} Tr\left(U^{-1}\partial_M U U^{-1}\partial_N U U^{-1}\partial_P U\right), \tag{34}$$

where $U = e^{\Phi}$. Defining $U_t = e^{t\Phi}$ and noting that

$$\frac{1}{3}\epsilon^{MNP}\partial_t Tr\left(U_t^{-1}\partial_M U_t U_t^{-1}\partial_N U_t U_t^{-1}\partial_P U_t\right) = \epsilon^{MNP}\partial_M\left(U_t^{-1}\partial_t U_t U_t^{-1}\partial_N U_t U_t^{-1}\partial_P U_t\right) \tag{35}$$

and that $U_t^{-1}\partial_t U_t = \Phi$, we can express (34) as a pure boundary term

$$-\frac{k_R}{2\pi}\int d^2z \int_0^1 dt\ Tr\left(\Phi\left[U_t^{-1}\partial_z U_t, U_t^{-1}\partial_{\bar{z}} U_t\right]\right)\Big|_{x^0=0}. \tag{36}$$

(36) can be evaluated by expanding the integrand in a power series in t and carrying out the t integral explicitly at every order. The final result for the full action is:

$$\begin{aligned} I[\mathbf{A}_z^{(0)}] = &\frac{k_R}{4\pi^2}\int d^2z d^2w\, (\bar{z}-\bar{w})^{-2}\, A_z^{(0)a}(\vec{z})\, A_z^{(0)a}(\vec{w}) \\ &-\frac{k_R}{12\pi^3}\epsilon^{abc}\int d^2z d^2w d^2v\, (\bar{z}-\bar{w})^{-1}(\bar{w}-\bar{v})^{-1}(\bar{v}-\bar{z})^{-1}\, A_z^{(0)a}(\vec{z}) A_z^{(0)b}(\vec{w}) A_z^{(0)c}(\vec{v}) \\ &+\cdots. \end{aligned} \tag{37}$$

Eqs. (26) and (37) now give:

$$\langle \bar{J}^{a_1}(\bar{z}_1)\bar{J}^{a_2}(\bar{z}_2)\rangle = \frac{k_R}{2}\,\delta_{a_1a_2}\,(\bar{z}_1-\bar{z}_2)^{-2}\,, \tag{38}$$

$$\langle \bar{J}^{a_1}(\bar{z}_1)\bar{J}^{a_2}(\bar{z}_2)\bar{J}^{a_3}(\bar{z}_3)\rangle = -\frac{ik_R}{2}\,\epsilon^{a_1a_2a_3}(\bar{z}_1-\bar{z}_2)^{-1}(\bar{z}_2-\bar{z}_3)^{-1}(\bar{z}_3-\bar{z}_1)^{-1}\,, \tag{39}$$

which are the expected conformal field theory correlation functions. Following this procedure we can in principle calculate arbitrary correlation functions of the SU(2) currents.

Let us now consider the effect of including additional gauge invariant terms in the action. Since such terms are functions of gauge field strength, the solution (28) satisfying $\mathbf{F}_{MN}=0$ continues to be solution of the equations of motion of the new theory. Furthermore the on-shell action is not modified since all the additional terms vanish when gauge field strength vanishes. As a result the correlation functions of the currents computed via (26) also remains unchanged. Thus we see that the vanishing of possible corrections to the correlators of SU(2) currents is intimately related to the vanishing of the additional terms in the action for an on-shell field configuration of the original theory. The latter in turn implies that the additional terms in the action can be removed by field redefinition.

We can now turn this argument around to see why the action in the gravity sector is also not renormalized. The non-renormalization of the boundary S-matrix (which in turn follows from supersymmetry relating the correlators of the currents and the stress tensor in the CFT) implies that any additional contribution to the action must vanish when original equations of motion are satisfied. This in turn implies that such additional terms can be removed by field redefinition. We emphasize that supersymmetry is crucial for this argument. In absence of supersymmetry relating the current correlators to the stress tensor correlators there is no reason for the latter to be not renormalized. This in turn would then imply that the effective action can receive corrections which do not vanish when the original equations of motion are satisfied. Hence such corrections cannot be removed by field redefinition.

References

[1] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [hep-th/9204099].

[2] A. Strominger, *Black hole entropy from near-horizon microstates*, *JHEP* **02** (1998) 009 [hep-th/9712251].

[3] J.D. Brown and M. Henneaux, *Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity*, *Commun. Math. Phys.* **104** (1986) 207.

[4] H. Saida and J. Soda, *Statistical entropy of BTZ black hole in higher curvature gravity*, *Phys. Lett.* **B 471** (2000) 358 [gr-qc/9909061].

[5] P. Kraus and F. Larsen, *Microscopic black hole entropy in theories with higher derivatives*, *JHEP* **09** (2005) 034 [hep-th/0506176].

[6] P. Kraus and F. Larsen, *Holographic gravitational anomalies*, *JHEP* **01** (2006) 022 [hep-th/0508218].

[7] S.N. Solodukhin, *Holography with gravitational Chern-Simons*, *Phys. Rev.* **D 74** (2006) 024015 [hep-th/0509148].

[8] B. Sahoo and A. Sen, *BTZ black hole with Chern-Simons and higher derivative terms*, *JHEP* **07** (2006) 008 [hep-th/0601228].

[9] P. Kraus, *Lectures on black holes and the $AdS_3/CFT(2)$ correspondence*, hep-th/0609074.

[10] Y. Tachikawa, *Black hole entropy in the presence of Chern-Simons terms*, *Class. and Quant. Grav.* **24** (2007) 737 [hep-th/0611141].

[11] S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys. (NY)* **140** (1982) 372[*Erratum ibid.* **185** (1988) 406] [*Ann. Phys. (NY)* **281** (2000) 409]; *Three-dimensional massive gauge theories*, *Phys. Rev. Lett.* **48** (1982) 975.

[12] J.M. Maldacena and A. Strominger, *AdS_3 black holes and a stringy exclusion principle*, *JHEP* **12** (1998) 005 [hep-th/9804085].

[13] R.M. Wald, *Black hole entropy in the Noether charge*, *Phys. Rev.* **D 48** (1993) 3427 [gr-qc/9307038].

[14] T. Jacobson, G. Kang and R.C. Myers, *On black hole entropy*, *Phys. Rev.* **D 49** (1994) 6587 [gr-qc/9312023].

[15] V. Iyer and R.M. Wald, *Some properties of Noether charge and a proposal for dynamical black hole entropy*, *Phys. Rev.* **D 50** (1994) 846 [gr-qc/9403028].

[16] T. Jacobson, G. Kang and R.C. Myers, *Black hole entropy in higher curvature gravity*, gr-qc/9502009.

[17] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[18] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[19] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[20] A. Dabholkar, A. Sen and S.P. Trivedi, *Black hole microstates and attractor without supersymmetry*, *JHEP* **01** (2007) 096 [hep-th/0611143].

[21] K. Hanaki, K. Ohashi and Y. Tachikawa, *Supersymmetric completion of an R^2 term in five-dimensional supergravity*, *Prog. Theor. Phys.* **117** (2007) 533 [hep-th/0611329].

[22] A. Castro, J.L. Davis, P. Kraus and F. Larsen, *5D attractors with higher derivatives*, *JHEP* **04** (2007) 091 [hep-th/0702072].

[23] A. Castro, J.L. Davis, P. Kraus and F. Larsen, *5D black holes and strings with higher derivatives*, hep-th/0703087.

[24] M. Alishahiha, *On R^2 corrections for 5D black holes*, hep-th/0703099.

[25] M. Nishimura and Y. Tanii, *Super Weyl anomalies in the AdS/CFT correspondence*, *Int. J. Mod. Phys.* **A 14** (1999) 3731 [hep-th/9904010].

[26] J.R. David, *Anti-de Sitter gravity associated with the supergroup* $\mathrm{SU}(1,1|2)\times\mathrm{SU}(1,1|2)$, *Mod. Phys. Lett.* **A 14** (1999) 1143 [hep-th/9904068].

JHEP07(2007)058

[27] A. Achucarro and P.K. Townsend, *A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories*, *Phys. Lett.* **B 180** (1986) 89.

[28] A. Achucarro and P.K. Townsend, *Extended supergravities in $D = (2 + 1)$ as Chern-Simons theories*, *Phys. Lett.* **B 229** (1989) 383.

[29] A. Giacomini, R. Troncoso and S. Willison, *Three-dimensional supergravity reloaded*, *Class. and Quant. Grav.* **24** (2007) 2845 [hep-th/0610077].

[30] E. Witten, *Quantum field theory and the Jones polynomial*, *Commun. Math. Phys.* **121** (1989) 351.

[31] E. Witten, $(2 + 1)$*-dimensional gravity as an exactly soluble system*, *Nucl. Phys.* **B 311** (1988) 46.

[32] P. Kraus and F. Larsen, *Partition functions and elliptic genera from supergravity*, *JHEP* **01** (2007) 002 [hep-th/0607138].

[33] S. Elitzur, G.W. Moore, A. Schwimmer and N. Seiberg, *Remarks on the canonical quantization of the Chern-Simons-Witten theory*, *Nucl. Phys.* **B 326** (1989) 108.